A GENERALISED MODEL FOR SUBMESOSCALE FRONTGENESIS

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The most prominent features are the meandering Gulf Stream jet/front system and the mesoscale (10-300 km) eddies produced via baroclinic instability of this current. However, closer examination shows a wealth of smaller scale eddies, particularly on the submesoscale (1-10 km).

The circulation associated with both the meso- and submesoscale eddies gives rise to strong horizontal convergence in regions between the eddies. This convergence drives the amplification of existing horizontal density gradients in a process known as frontogenesis, as studied by Hoskins and Bretherton [HB].

The dynamical difference between frontogenesis at the meso- and submesoscales is encapsulated by the non-dimensional strain \( \delta \) (or Rossby number) associated with the eddies:

\[
\delta = \frac{1}{\pi} \frac{|b_1|}{\pi \delta} = \frac{\alpha}{\pi} \tag{1}
\]

The strain \( \delta \) is typically \( O(0.1) \) on the mesoscale, but is substantially larger (\( O(1) \)) on the submesoscale. HB’s model assumes small strain, \( \delta \ll 1 \), and so is not applicable in describing frontogenesis between submesoscale eddies.

Here we describe a new generalised model that is valid for \( O(1) \) strains.

A Simple Model

Consider an initially weak front between a pair of eddies, as represented in the schematic above. The front is in an initial state of balance with an along front jet. Apart from the strain \( \delta \), there are two other dimensionless parameters relevant to the system: (1) The Burger number,

\[
Bu = \frac{N H}{f L} \tag{2}
\]

where \( N \) is the along-front velocity, with \( v \) the geostrophic part, \( b_1 \) the buoyancy, and \( y \) the potential vorticity (PV). Equation (5) is the combined horizontal momentum equation which takes the form of a Lagrangian harmonic oscillator, forced by the geostrophic jet \( v_0 \). We seek the (forced) solution to this equation, eliminating the explicit wave solutions associated with initial conditions. Boundary conditions on the buoyancy, associated with the initial surface buoyancy gradient, are imposed through the thermal wind equation (6). The buoyancy \( b_1 \) is related to the velocity field \( v \) through the conservation of PV (7). The problem is then solved numerically.

To solve the system of equations we introduce a new coordinate, \( X \), called the Generalised Momentum Coordinate,

\[
X = \eta \left( x + \frac{b_1}{Bu} \right) = \eta \delta X \tag{8}
\]

where \( X \) is the usual momentum coordinate, as used by HB. The generalised coordinate has the special property of being materially conserved (i.e. following the flow) for any value of strain.

\[
\frac{D X}{D t} = 0 \tag{9}
\]

Writing (5, 6, 7) in this coordinate, an analytic solution may be obtained in the limit:

\[
Ro^2 \delta^2 \ll (1 + \delta^2 + Bu^2 \delta^2)^2 \tag{10}
\]

The HB model, which requires \( \delta \ll 1 \), is a subset of this limit. However, our model is more general in that the strain \( \delta \) may be large if the Rossby number \( Ro \) is sufficiently small.

Greens Functions

Consider placing an impulse of along-front momentum at the origin of the momentum coordinate \( X \) axis, in the centre of the strain flow. The corresponding steady adjusted states for velocity \( v \) are shown below, as predicted by the generalised (solid) and HB (dashed) models, for a strain of \( \delta = 0.9 \), \( Ro = 0.6 \) and \( Bu = 1.5 \) (as in the Large Strain solution shown on the right).

The impulse response from the HB model extends smoothly out to infinity. In contrast, the impulse response from the generalised model is confined to a finite region about the origin. This difference is due to the generalised model incorporating the effect of strain on the propagation of waves in the system: waves are responsible for spreading the momentum impulse outwards from the origin. However, waves can only propagate where their outward group velocity exceeds the incoming flow speed, \( v_0 > \alpha \). The impulse response is therefore confined to the region

\[
|X| < \frac{b_1}{Bu} \tag{11}
\]

for the \( n=1 \) vertical mode.

The impulse responses \( v_1(X, Z) \) shown in the figure above are the Greens functions for the problem, from which the fully time-dependent solution may be constructed:

\[
v(X, Z, T) = \int_{-\infty}^{\infty} v_1(X - X_p, Z) \frac{\partial}{\partial X_p} (X_p \epsilon e^{iT}) \, dX_p \tag{12}
\]

where \( b_1(X) \) is the imposed boundary profile of buoyancy, the gradient of which is amplified by the strain flow. The other velocity and buoyancy fields are obtained via spatial/temporal derivatives/integrals of \( v \).

REFERENCES


Small Strain: \( \delta = 0.2, Ro = 0.6, Bu = 1.5 \)

The squeezing of the front by the convergent strain ultimately drives the formation of the discontinuity in the velocity and buoyancy fields on the boundary. The vertical velocity fields just prior to this time arising from the generalised model, HB model and a fully non-linear numerical simulation are shown below. Buoyancy contours are superimposed in grey.

Wave Amplitude

The amplitude of the spontaneously generated waves depends on the magnitude of the strain \( \delta \). The relative integrated amplitude of the wave streamfunction compared to the net circulation, as predicted by the generalised model, is shown below. Wave generation is exponentially small for strains less than about 0.2, but finite for larger strains. Note that the separation of the flow into wave and mean parts breaks down as \( \delta \to 1 \) (see equation 5).