1. Introduction

Context:
- Oceanic fronts: horizontal boundaries between water masses (e.g., Gulf Stream separating sub-polar from sub-tropical waters)
- Oceanic fronts characterized by:
  * strong lateral density gradient, thermal wind shear,
  * strong ageostrophic, vertical motions, enhanced turbulence,
  * strong internal wave activity.
- Understanding frontal mixing: crucial to understand air-sea exchanges => climate modeling, biology…

Internal waves in fronts:
- Peculiar properties due to slanted isopycnals (Whitt & Thomas 2013).
- Can “classical” (flat isopycnals) internal wave physics give insight about “frontal” internal wave physics?

How are the reflection properties of internal waves modified by the presence of an oceanic front?

2. Critical, forward and backward reflections

- Oceanic fronts characterized by strong lateral density gradients: $S' = (\rho/\rho_0)(d\rho/\rho_0)dz$
- Consequence on internal waves: unusual dispersion relationship: $\psi(j\beta) = f(N^2 + f^2)^{-1/2}$
- The slope of wave phase lines are symmetric around the isopycnal slope (if non-hydrostatic): $\beta = (\text{sink}) - S/N^2 \pm (\omega f/N^2)(\omega f/N^2)$
- For $\omega = f$, critical reflection against the ocean surface: $\beta = 0$.
- Similar to classical internal waves reflecting onto a slope, frontal internal waves reflecting onto the ocean surface can experience critical reflection for $\omega = f$.
- If $\omega > f$: “forward” (sub-critical) reflection: if $\omega < f$: “backward” (super-critical) reflection.

3. Set-Up

- Two-dimensional ($x$, $z$) simulations,
  - $\eta = \Delta x = 1.56 \text{ m, } \Delta z = 0.77 \text{ cm}$
  - $N^2 = 10^{-2} \text{ s}^2$, $f = 10^{-5} \text{ s}$
  - Geostrophic Richardson: $Ri = f^2/N^2 = 1.05$
  - Background PV: $f(N^2 - 1/Ri) > 0$
  - Waves forced in the volume (cf. Figure), minimal generation of PV
  - Forcing amplitude tuned such that incident wave has Richardson number $Ri$ when reaching the surface
  - Free-slip, rigid lids on top & bottom, periodic in $x$.

Equations solved by the code (Winters et al. ’04):

\begin{equation}
\begin{align*}
\omega &+ f \hat{z} \times \hat{u} = [S'(f) \psi \beta + b \cdot \nabla \times \psi \beta + \nabla P] D_{\psi}\beta,
\end{align*}
\end{equation}

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D_{\psi} \beta = f \hat{u} + S' w + N^2 w + c \nabla D_{\psi} \beta = D_{\psi} \beta : \quad u_1 + w_1 = 0, \\
\epsilon = 0 \text{ or } 1
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4. Linear reflections

Forward reflection ($\omega > f$)

- Unsurprising result: reflection along characteristics, viscous decay.

Backward reflection ($\omega < f$)

- No backward reflection! Wave entirely absorbed under the surface.

Critical reflection ($\omega = f$)

Horizontal velocity field ($u$, mm/s):

- Structure of sub-surface flow governed by:

\begin{equation}
\psi_x + 2i \psi_y - 2ik \psi_x - \frac{2ki}{f} \psi_y - \frac{k^2}{f} \psi_x - \frac{\psi}{f} \psi_y = 0,
\end{equation}

with $\psi = \psi(y) \exp(ik \xi)$, $\beta \in \mathbb{R}$. Make possible because $\omega = f$ and $\beta \approx k_{\text{inert}}$ for linear reflections.

5. Non-linear, critical reflection ($\omega_{\text{forcing}} = f$)

Snapshots:

- (a) $u$ (mm/s). Solid: isopycnals. Dashed: wave forcing position.
- (b) $\psi$ (mm/s).
- (c) $\psi$ (mm/s). Lines: passive tracer contours.

Non-linear flow active well below the surface.

Local extrema of $\langle \psi \rangle$ for $R_i = 10$ (black) and $R_i = 100$ (gray), for $\psi_x = 2 \text{ mm}^2/\text{s}$ (circles) and $\psi_y = 4 \text{ mm}^2/\text{s}$ (crosses).

- Top $2.5 \text{ m}$: strong decay of the sub-surface flow, sensitive to amplitude, less so to viscosity.
- Below $z = 2.5 \text{ m}$: smaller decay of the sub-surface flow, sensitive to viscosity, less so to amplitude.

Link with the linear theory: unclear at this point...

Transition to turbulence: normalized frequency spectra of $u$ for $R_i = 3$ (black), $R_i = 1$ (dark) and $R_i = 0.3$ (light).

Increased amplitude $\Rightarrow$ stable harmonics disappear, flow becomes turbulent. Well-known in the classical case.

6. Non-linear, non-critical reflections ($\omega_{\text{forcing}} \neq f$)

- No apparent reflected wave.

7. Energetics

- Ratio, averaged over $x$ and time, of:

\begin{equation}
\begin{align*}
\text{Kinetic energy dissipation} & \div \text{top } 15 \text{ m}, \text{over } 2 \text{ days}, \\
\text{Incident kinetic flux} (\text{PW}) & \div \text{top } 15 \text{ m}.
\end{align*}
\end{equation}

- Forward reflections $\Rightarrow$ deep energy propagation.
- Around $\omega = f$: more energy is dissipated that is supplied by the incident wave!
- Geostrophic flow supplies energy to the ageostrophic flow.

Reflecting near-$f$ waves can potentially drain energy out of fronts (in the absence of surface forcing).

Absent in classical reflections, this effect is a genuine feature of the frontal case!

8. Conclusions

- In fronts, inertial waves experience critical reflections against the ocean surface.
- Linear reflection properties are governed by viscous theory, although more complicated than mere Ekman layer dynamics.
- Non-linear, critical reflection: ageostrophic energy present well below the surface. This flow is entirely forced, no radiation of freely-propagating waves.
- Non-linear, backward reflection: wave absorbed under the surface, no apparent reflection.
- Non-linear, forward reflection: wave reflects, non-linear interactions happen, reflections and harmonics propagate energy deep down.
- Reflecting near-$f$ waves can potentially drain energy out of fronts, in the absence of surface forcing.
- Multiple avenues for transfer of knowledge from classical internal wave science to frontal and sub-mesoscale dynamics: 3D effects, wave-mean flow interactions, turbulence and mixing...

References:

\[ \hat{x} \cdot k_{\text{incident}} = \hat{x} \cdot k_{\text{reflected}} \]